

Locality of quantum entanglement

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Abstract

This article presents a local realistic interpretation of quantum entanglement. The entanglement is explained as innate interference between the non-empty state associated with the peaked piece of one particle and the empty states associated with the non-peaked pieces of the others of entangled particles, which inseparably join together. The correlation of the results of measurements on the ensemble of composite entangled systems is related to this kind of interference. Consequently, there is no nonlocal influence between entangled particles in measurements. Particularly, this explanation thus rules out the possibility of quantum teleportation which is nowadays considered as one of cornerstones of quantum information processing. Besides, likewise, communication and computation schemes based on alleged spooky action at a distance are unlikely to be promising.

1 Introduction

Recently, the application of quantum mechanics enters the field of information science and technology. Some current theories of quantum information seem to be dependent on interpretation of quantum mechanics. The local and nonlocal interpretations of quantum mechanics lead to different consequences. For example, the latter leads to the idea of quantum teleportation. Quantum teleportation refers to the scenario that a quantum state has disappeared from a location and repeated instantaneously in a distant region. Bennett *et al.* in 1993 claimed: “An unknown quantum state $|\phi\rangle$ can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlation.” [1] However Einstein *et al.* (EPR) denied existence

of any nonlocal influences between spacelike separated particles and concluded that the quantum-mechanical description is incomplete. At the beginning of their paper [2], they stated: “Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.” And at its end they concluded: “While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question whether or not such a description exists. We believe, however, that such a theory is possible.” They did not mention any types of hidden variables in their paper. Dirac said: “I think that it is quite likely that at some future time we may get an improved quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view.” [3] Unfortunately, the EPR argument has been considered as a failure by the majority of physicists. In favor of Einstein’s local realism the present author will give a local realistic interpretation of quantum entanglement and shows the impossibility of quantum teleportation.

2 Locality of quantum entanglement

The explanation of entanglement of a composite quantum system is fundamentally related to the description of a single system. In Ref.[4] the author describes a free particle as linear non-spreading wave packet in the framework of special relativity and logically explains quantum interference experiments. The wave packet consists of countless Fourier components among which there is one called characteristic component which frequency and wave vector are related to the energy and momentum of the particle, respectively. The Schrödinger wave packet or light pulse consists of such characteristic components which are related to different energies and momenta and thus spreads in principle. The non-spreading wave packet contains a peak where the phases of all the components are same and its off-peak part where the distribution of their phases makes the resulting amplitude nearly infinitesimal. Diffraction and interference phenomena demonstrate that cutting a piece away from the off-peak part or recombining it will change the path of the peak and/or the energy of the particle. Different from classical reality, a quantum reality such as photon, electron or atom, has to be considered to contain a peak and its off-peak part which plays an essential role in quantum interference and is the root of quantum weirdness. The concept of a point-like classical particle

reflects the ignorance of the existence of the off-peak part.

In a composite system containing two or more particles in a superposition state, according to Schrödinger [5], there exists entanglement. From the concept of the non-spreading wave packet as being quantum reality, the entanglement of a pair of particles can be explained as innate interference between the peaked piece of one particle and the non-peaked piece of the other of the pair, which inseparably join together. Yet the two peaked pieces keep mutual independence if the two peaks are spacelike separated. That is, quantum entanglement is local and cannot be regarded as “spooky action at a distance” (Einstein’s words [6], March 1947).

For example, we consider a pair of entangled particles described by the unnormalized wave function

$$\Psi_{p_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(e^{-ip_0x_1/\hbar} \cdot e^{ip_0x_2/\hbar} + e^{-ip_0x_2/\hbar} \cdot e^{ip_0x_1/\hbar}) \quad (1)$$

which is interpreted as a probability amplitude. In order to make quantum-mechanical description formally more complete, let’s use the symbol $\# \dots \#$ to denote the function which is the characteristic component associated with the non-peaked piece of a quantum system. We will call the state indicated by this symbol “empty state” which carries information on the non-peaked piece. So the real state of the pair of particles should be assumed to be one of the following two:

$$\Psi_{p_0, -p_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(e^{-ip_0x_1/\hbar} \cdot e^{ip_0x_2/\hbar} + \#e^{-ip_0x_2/\hbar} \# \cdot \#e^{ip_0x_1/\hbar} \#) \quad (2)$$

$$\Psi_{-p_0, p_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(\#e^{-ip_0x_1/\hbar} \# \cdot \#e^{ip_0x_2/\hbar} \# + e^{-ip_0x_2/\hbar} \cdot e^{ip_0x_1/\hbar}) \quad (3)$$

These equations show that there is innate interference between the non-empty state associated with the peaked piece of one particle and the empty state associated with the non-peaked piece of the other of the pair, which inseparably join together. Consequently, the correlation of results of measurements on the ensemble of the pairs is related to this kind of interference. So the correlation characterizing quantum entanglement is local.

Now we consider the case in the position representation where a pair of particles is in the entangled state

$$\Psi_{x_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(\delta(x_0 - x_1) \cdot \delta(x_0 + x_2) + \delta(x_0 - x_2) \cdot \delta(x_0 + x_1)) \quad (4)$$

The real state of the pair is one of the following two:

$$\Psi_{x_0, -x_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(\delta(x_0 - x_1) \cdot \delta(x_0 + x_2) + \# \delta(x_0 - x_2) \# \cdot \# \delta(x_0 + x_1) \#) \quad (5)$$

$$\Psi_{-x_0, x_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(\# \delta(x_0 - x_1) \# \cdot \# \delta(x_0 + x_2) \# + \delta(x_0 - x_2) \cdot \delta(x_0 + x_1)) \quad (6)$$

Eq.4 implies that after the position measurement on both particles, particle 1 and 2 will have been found simultaneously at $x_1 = x_0$ and $x_2 = -x_0$ or at $x_1 = -x_0$ and $x_2 = x_0$, or implies equivalently in the statistical sense that particle 1 of a pair expressed by Eq.5 and particle 2 of another pair expressed by Eq.6 in their pair ensemble will have been found simultaneously at $x_1 = x_2 = x_0$ with equal probability after the position measurement at that place.

Furthermore, in order to discuss measurement effects on the entangled pair we consider the case where the two particles of the entangled pair described by Eq.1 are spacelike separated. If only particle 1 of the pair has been measured and found at $x_1 = x_0$, the pair will have jumped into the real state

$$\Psi_{x_0, -p_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(\delta(x_0 - x_1) \cdot e^{ip_0 x_2 / \hbar} + \# \delta(x_0 - x_2) \# \cdot \# e^{ip_0 x_1 / \hbar} \#) \quad (7)$$

or if only particle 2 has been measured and found at $x_2 = -x_0$, the pair will have jumped into the real state

$$\Psi_{p_0, -x_0}(x_1, x_2) = \frac{1}{\sqrt{2}}(e^{-ip_0 x_1 / \hbar} \cdot \delta(x_0 + x_2) + \# e^{-ip_0 x_2 / \hbar} \# \cdot \# \delta(x_0 + x_1) \#) \quad (8)$$

These equations show that there is no nonlocal influence between the two entangled particles in the measurement. Let's explain it in more details. Assuming that Alice locates on the positive half-axis of x and Bob on the negative one, if she has found particle 1 at $x_1 = x_0$ after her measurement using a tool such as a clamp equivalent to an extremely narrow and deep potential well, the state $e^{-ip_0 x_1 / \hbar}$ of the particle will have evolved into $\delta(x_1 - x_0)$ which is an eigenstate of the position operator X , while the state $e^{ip_0 x_2 / \hbar}$ of particle 2 remains intact since the two associated peaks are spacelike separated. On the other hand, the empty state $\# e^{-ip_0 x_2 / \hbar} \#$ of particle 2 will also have evolved simultaneously into the empty state $\# \delta(x_2 - x_0) \#$ because of action of the clamp, while the empty state $\# e^{ip_0 x_1 / \hbar} \#$ of particle 1 remains intact. It is similar for the case where Bob performs a position measurement on particle 3 in the same way. So quantum entanglement has no nonlocality feature. Thus the Bell inequalities [7] are not the

touchstones for judging whether quantum mechanics is local or nonlocal. It is a mistake that the Bell inequality violation is interpreted as evidence for nonlocal influences in the measurement on entangled particles. We see that containing a return to determinism by supplementing the non-peaked piece of a system as a new kind of quantum reality, the improved quantum mechanics yet completely tallies predictions for correlation measurement led by quantum entanglement. We are aware that quantum mechanics does not explicitly require and yet not implicitly rule out this kind of supplementary reality. However, the proper interpretation of quantum mechanics exactly needs the supplement as a basis.

3 Impossibility of quantum teleportation

Now we are in position to discuss whether or not quantum teleportation is possible. According Bennett *et al.* [1], if Alice and Bob share the entangled state of a pair of particles with spin- $\frac{1}{2}$

$$|\Psi_{23}^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_3\rangle|\downarrow_2\rangle) \quad (9)$$

Alice can send Bob an unknown arbitrary state

$$|\phi_1\rangle = \alpha|\uparrow_1\rangle + \beta|\downarrow_1\rangle, |\alpha|^2 + |\beta|^2 = 1 \quad (10)$$

by making use of the following direct product state

$$\begin{aligned} |\Psi_{123}\rangle &= |\phi_1\rangle|\Psi_{23}^-\rangle = \frac{1}{2} [|\Psi_{12}^+\rangle(-\alpha|\uparrow_3\rangle + \beta|\downarrow_3\rangle) + |\Psi_{12}^-\rangle(-\alpha|\uparrow_3\rangle - \beta|\downarrow_3\rangle) \\ &\quad + |\Phi_{12}^+\rangle(-\beta|\uparrow_3\rangle + \alpha|\downarrow_3\rangle) + |\Phi_{12}^-\rangle(\beta|\uparrow_3\rangle + \alpha|\downarrow_3\rangle)] \\ &= \frac{1}{2} [|\Psi_{12}^+\rangle M_{11}|\phi_3\rangle + |\Psi_{12}^-\rangle M_{00}|\phi_3\rangle + |\Phi_{12}^+\rangle M_{10}|\phi_3\rangle + |\Phi_{12}^-\rangle M_{01}|\phi_3\rangle], \\ &\quad |\phi_3\rangle = \alpha|\uparrow_3\rangle + \beta|\downarrow_3\rangle \end{aligned} \quad (11)$$

where the set

$$|\Psi_{12}^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle \pm |\uparrow_2\rangle|\downarrow_1\rangle), |\Phi_{12}^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\uparrow_2\rangle \pm |\downarrow_2\rangle|\downarrow_1\rangle) \quad (12)$$

is the so-called bell basis, and

$$M_{00} = -I, M_{01} = \sigma_x, M_{10} = -i\sigma_y, M_{11} = -\sigma_z \quad (13)$$

in which I is the identity operator and $\sigma_x, \sigma_y, \sigma_z$ are the Pauli operators:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (14)$$

With the generally accepted viewpoint, after Alice's Bell-basis measurement on particle 1 and 2, Bob's particle 3 will have collapsed into a state $M_{i'j'}|\phi_3\rangle$, namely one of the four states $M_{ij}|\phi_3\rangle, i, j = 0, 1$. Consequently, in their opinion, if Alice communicates through a classical channel to Bob 2-bit information (i', j') representing the result of the measurement, Bob "can construct an accurate replica $|\phi_1\rangle$ " by using the inverse operation $M_{i'j'}^{-1}$ on his $M_{i'j'}|\phi_3\rangle$. So the state is teleported in such a way from Alice to Bob with infinite precision (i.e. infinite amount of data) only at a cost of 2-bit information. Yet it is said that such a cheap transmission of information is warranted by quantum mechanics principle. Quantum teleportation is nowadays considered as one of cornerstones of quantum information processing.

However, according to the above local realistic interpretation, before Alice's Bell-basis measurement, particle 3 in Bob's hands is in one of the following two real states:

$$|\Psi_{23}^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow_2\rangle|\downarrow_3\rangle - \#|\uparrow_3\rangle\#\#|\downarrow_2\rangle\#) \quad (15)$$

$$|\Psi_{23}^-\rangle = \frac{1}{\sqrt{2}}(\#|\uparrow_2\rangle\#\#|\downarrow_3\rangle\# - |\uparrow_3\rangle|\downarrow_2\rangle) \quad (16)$$

After her measurement, on her side the empty state $\#|\uparrow_3\rangle\#$ or $\#|\downarrow_3\rangle\#$ of particle 3 will have evolved into one of the four empty states $M_{ij}(\alpha\#|\uparrow_3\rangle\# + \beta\#|\downarrow_3\rangle\#), i, j = 0, 1$, while on Bob's side the non-empty state $|\uparrow_3\rangle$ or $|\downarrow_3\rangle$ of particle 3 remains intact. In reality, her measurement only influences the state $|\phi_1\rangle$ of particle 1 and both the inseparable interfering states, namely the non-empty state of particle 2 and the empty state of particle 3 on the side of herself. Thus the local realistic interpretation rules out the possibility of quantum teleportation. We argue that the idea of quantum teleportation is based on the misunderstanding of quantum mechanics and is invalid. Besides, likewise, communication and computation schemes based on alleged spooky action at a distance are unlikely to be promising. Recently, a lot of researchers have spent much effort to do study of realization of quantum teleportation [8-15]. Sorry, to the best of the author's knowledge, until now there seems to be, however, no experiment that deserves to be called accomplishing successful quantum teleportation and checks definitely its possibility.

4 Conclusion

This article presents a local realistic interpretation of quantum entanglement. The entanglement is explained as innate interference between the non-empty state associated with the peaked piece of one particle and the empty states associated with the non-peaked pieces of the others of entangled particles, which inseparably join together. The correlation of the results of measurements on the ensemble of composite entangled systems is related to this kind of interference. Consequently, there is no nonlocal influence between entangled particles in measurements. Particularly, this explanation thus rules out the possibility of quantum teleportation which is nowadays considered as one of cornerstones of quantum information processing. Besides, likewise, communication and computation schemes based on alleged spooky action at a distance are unlikely to be promising.

References

- [1] Bennett, C. H., *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
- [2] Einstein, A., Podolsky, B. and Rosen, N., Phys. Rev. **47**, 777 (1935).
- [3] Dirac, P. A. M., “The development of quantum mechanics ”, in Directions in Physics, eds. H. Hora and J. R. Shepanski, (Wiley, New york, 1978), p. 10.
- [4] Wang Guowen, Heuristic explanation of quantum interference experiments, at http://arxiv.org/PS_cache/quant-ph/pdf/0501/0501148.pdf
- [5] Schrödinger, E., Proc. Cam. Phil. Soc., **31**, 555, (1935).
- [6] Einstein, A., in Born-Einstein Letters, transl. I. Born, (Walker, New York, 1971), p. 158.
- [7] Bell, J. S., Physics, **1**, 195 (1964).
- [8] Bouwmeester, D. *et al.*, Nature, **390**, 575 (1997).
- [9] Furusawa, A. *et al.*, Science, **282**, 706 (1998).
- [10] Kim, Y-H., Kulik, S. P. and Shih, Y., Phys. Rev. Lett., **86**, 1370 (2001).
- [11] Pan, J. -W. *et al.*, Phys. Rev. Lett., **86**, 4435 (2001).

- [12] Marcikic, I. *et al.*, Nature, **421**, 509 (2003).
- [13] Zhang, T. C. *et al.*, Phys. Rev. A, **67**, 033802 (2003).
- [14] Zhi Zhao *et al.*, Nature, **430**, 54 (2004).
- [15] Ursin, R. *et al.*, Nature, **430**, 849 (2004).